On Chern-Weil theory of groups preserving a contact form

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Example: the unit sphere $S^{2n+1} \subseteq \mathbb{R}^{2n+2} = \mathbb{C}^{n+1}$

- the standard contact structure $\xi_{\rm std}=TS^{2n+1}\cap i(TS^{2n+1})$ ("complex tangencies")
- if $(x_1, y_1, \dots, x_{n+1}, y_{n+1})$ are the euclidean coordinates on \mathbb{R}^{2n+1} , then a contact form is

$$\alpha_{\mathsf{std}} = \sum_{j=1}^{n+1} (x_j dy_j - y_j dx_j)|_{TS^{2n+1}}.$$

Let $(M, \xi = \ker \alpha)$ be a contact manifold.

• the group of contactomorphisms

$$\operatorname{Cont}(M,\xi) = \{ f \in \operatorname{Diff}(M) \mid Tf(\xi) = \xi \} =$$

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I study how $(M, \xi = \ker \alpha)$ can fibre over other manifolds in a (strictly) contact way.



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 $\operatorname{Cont}(M,\alpha)$ acts on $\operatorname{CFr} E$ from the right by composition of mappings and this action is free and transitive on fibres, so $\operatorname{CFr} E \to B$ is a **principal Cont** (M,α) -bundle ...



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Remark: Characteristic classes of principal G-bundles are in 1-1 correspondence with elements of $H^*(BG)$, the cohomology ring of the classifying space BG of the group G.



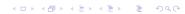
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Goal: describe $H^*(\mathrm{BCont}(M,\alpha))$ – characteristic classes for principal $\mathrm{Cont}(M,\alpha)$ -bundles / strictly contact fibre bundles with typical fibre (M,α)



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Then

$$R_{d\theta} : \Lambda^2 TP \times \ldots \times \Lambda^2 TP \xrightarrow{d\theta \times \ldots \times d\theta} \mathfrak{g} \times \ldots \times \mathfrak{g} \xrightarrow{R} \mathbb{R}$$

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is a 2k-form on P, which descends to a **closed** 2k-form $\widetilde{R}_{d\theta}$ on B. Its de Rham cohomology class $\chi_R(P) := [\widetilde{R}_{d\theta}] \in H^{2k}_{\operatorname{deR}}(B; \mathbb{R})$ is **independent of** the choice of the connection θ and so the assignment " $(P \to B) \mapsto \chi_R(P)$ " defines a characteristic class.



Example

• Let $G = GL(n, \mathbb{R})$, $\mathfrak{g} = \mathsf{Mat}(n; \mathbb{R})$. Consider the invariant polynomials $R_k(A_1, \ldots, A_k) = \mathsf{tr}(A_1 \cdot \ldots \cdot A_k)$, $k \in \mathbb{N}$.

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- ② Let $G = \operatorname{Cont}(M, \alpha)$. Its Lie algebra is the algebra of vector fields $X \colon M \to TM$ which satisfy $\mathcal{L}_X \alpha = 0$. Applying α , we get a function $\alpha(X) \colon M \to \mathbb{R}$.

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$$R_k(X_1,\ldots,X_k) = \int_{M} \alpha(X_1) \cdot \ldots \cdot \alpha(X_k) \alpha \wedge (d\alpha)^n$$

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Theorem (OS): Let $(M, \xi = \ker \alpha)$ be a closed connected **regular** contact manifold and let $\varphi \colon S^1 \to \operatorname{Cont}(M, \alpha)$ be the S^1 -action of the Reeb flow of α . Then the induced map on the cohomology of the classifying spaces

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Idea of proof: Start with a principal S^1 -bundle $P \to B$ and construct the associated bundle $E = P \times_{S^1} M \to B$ with fibre (M,α) . This will be a strictly contact bundle. Pull back the contact classes $\chi_k(E)$ to $P \to B$ and show that the pull-backs are the powers $e(P)^k$ of the Euler class of P.